

Cours G-Turinici "Méthodes numériques : évolution"

mars 2023

$$\delta X_{n+1} = \lambda_{n+1}$$

$$\frac{\partial F}{\partial x_n}$$

$$\delta X_{n+1}$$

$X_N := \underbrace{F}_{\frac{\partial F}{\partial X_N}}$ connue
 $\approx x(T)$ dépendance

$$x_0 = \underbrace{\frac{\partial F}{\partial x}}_{\frac{\partial F}{\partial x_0}} ?$$

$$\frac{\partial F}{\partial x(T)} = \frac{\partial F}{\partial x(0)}$$

$$x(T) \approx F(x(0))$$

est connue.

$$\text{Euler Explicit form } \dot{x}(t) = -\lambda(t) \frac{\frac{\partial F}{\partial x}(t, x(t), u(t))}{g(t)}$$

$$\lambda_n = \lambda_{n-1} + (-h)$$

$$\dot{x}(t) = -\lambda(t) g(t)$$

$$\sum h_i = \lambda(T-t)$$

$$\dot{x}(t) = -\lambda'(T-t) g(T-t)$$

$$\dot{x}(t) = \underbrace{\dot{x}(t)}_{\lambda(t)} g(T-t)$$

$$\tilde{x}_{n+1} = \tilde{x}_n + h \cdot \tilde{x}_n'(t_n)$$

$$\text{approx do } \tilde{x}(t_{n+1}) \approx \lambda(T-t_n)$$

$$\tilde{y}_{n+1} \approx \tilde{y}_n + h \sum_{k=n}^N f(t_k) \quad (\text{à})$$

$$\tilde{y}_n = \tilde{y}_{n-1} + h \sum_{k=n-1}^{N-1} f(t_k)$$

↓
approx de $\lambda(T - t_{n-1}) \sim \lambda \left(\frac{\sum_{k=n-1}^N f_k}{\sum_{k=n-1}^N 1} \right)$

approx de $\lambda(T - t_n)$

$$\lambda(N_n)h$$

$$\lambda(t_{n-1})$$

$$x(t_n)$$

$$t_k = \tilde{y}_n$$

$$\lambda_{k-1} = \tilde{x}_k + h \tilde{x}_k \frac{df}{dx}(t_k, x_k, u_k)$$

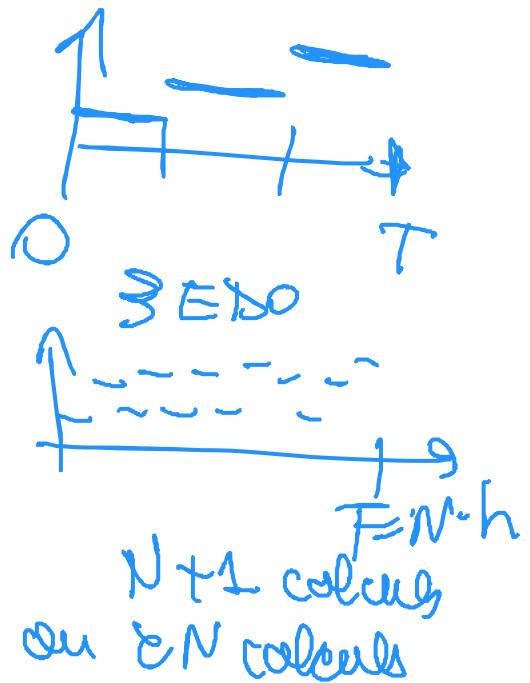
Euler Explicit
en

arrive en temps pour $\lambda' = -\lambda \frac{df}{dx}$

Donc $\lambda_n \approx \frac{df}{dx}$ est une discrétisation d'ES pour

l'équation rétrogradée à $\begin{cases} \lambda'(t) = -\lambda(t) \frac{df}{dx}(t, x(t), u(t)) \\ \lambda(T) = \frac{df(x(T))}{dx(t)} \end{cases}$

$$\frac{dF}{du(t)} = \lambda(t) \cdot \frac{df}{dx}(t, x(t), u(t))$$



$$G > G(X, \mu, \lambda)$$

$$\frac{\partial G}{\partial X(T)} = \frac{\partial F(X(T))}{\partial X(T)} + \lambda(T)$$

$$\frac{\partial G}{\partial X(T)} > 0 \Rightarrow \lambda(T) = -\frac{\partial F(X(T))}{\partial X(T)}$$

$$G(X, \mu, \lambda) = F(X(T)) - \int_0^T [X(t) - \bar{F}(t, \mu, \lambda)] \lambda(t) dt$$

$$= F(X(T)) - X \lambda \int_0^T + \int_0^T [\bar{X}(t) \lambda'(t) + \lambda \bar{F}'(t, \mu, \lambda)] dt$$

$$\lambda(T) = -\frac{\partial F}{\partial X(T)}$$

$$\lambda' + \lambda \bar{F} \geq 0 \quad \boxed{\lambda' = -\lambda \bar{F}}$$

$$\frac{\partial G}{\partial \mu} > 0 \Rightarrow \frac{\partial \bar{F}}{\partial \mu} > 0 \text{ si}$$

$$F(X(T)) \approx S(0) - \lambda(T) \epsilon(C) \quad X \in \mathbb{R}^n$$

calcul de $\frac{\partial}{\partial \beta(i)}$ F :

$$G \approx S(0) - \int_0^T C(\beta(t)) dt$$

$$- \int_0^T (\beta'(t) + \beta(i)) \lambda(t) dt - \int_0^T \mu(t) (i^T - \beta(t)) dt$$

$$R = I - S - I$$

$$\frac{\partial G}{\partial \beta} = \mu'(C) - \lambda \beta + \mu \beta i \quad \boxed{\beta}$$

$$\left. \begin{array}{l} \frac{\partial G}{\partial \lambda} : \lambda' = -\beta \bar{i} \\ \frac{\partial G}{\partial \mu} : \bar{i}' = \beta \bar{s} - \bar{i} \end{array} \right\} \text{et}$$

$$\frac{\partial G}{\partial \lambda} \approx 0 \quad \boxed{i^T - \beta \bar{i} + \mu \beta i = 0}$$

$$\frac{\partial G}{\partial \mu} \approx 0 \quad \boxed{-\beta \bar{s} \lambda + \mu' + \mu \beta s - \bar{s} \mu = 0}$$

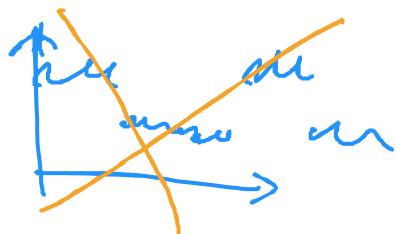
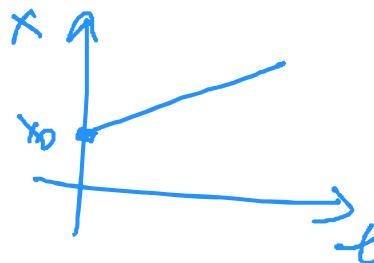
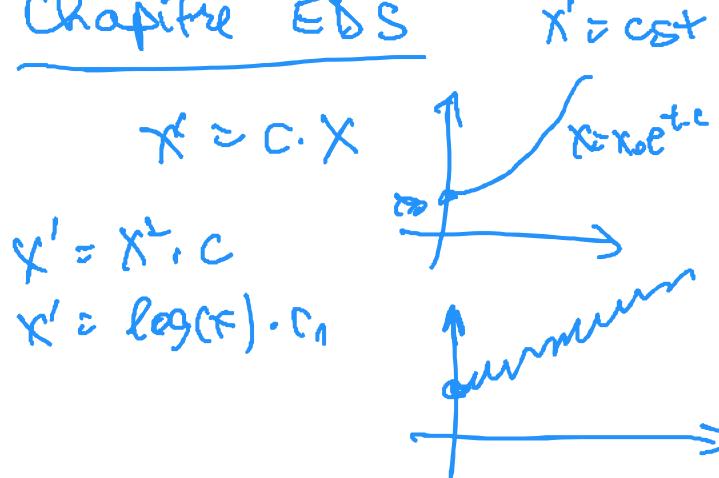
$$\frac{\partial G}{\partial \beta} \approx 0 \quad \boxed{-I + \lambda(T) \approx 0} \quad \boxed{\lambda(T) = 1}$$

$$\frac{\partial G}{\partial \beta(i)} \approx 0 \quad \boxed{\mu(i) \approx 0}$$

Ceci montre même si β n'est pas encore l'optimum.

- Donc pour le calcul de $\frac{\partial G}{\partial \beta}$:
- ① on calcule $\delta(t), i(t)$
 - ② on calcule $\lambda(t), \mu(t)$
 - ③ on applique (p.e.)

Chapitre EDS



\rightarrow Mouvements continus

↑ "stationnaire"

assez réguliers comme les

$B_t(w)$
 v.a. dép
 du temps
 $B(t, w)$
 $\Rightarrow t \rightarrow B(t, w)$



$$x'(t) = f(t, x(t)) + \text{bruit}$$

mouvements $\sim w$

$$x(t) = x_0 + \int_0^t f(s, x(s)) ds$$

$$t \quad B_t(w) \approx v.a.$$

B_t : prix
 H_t : quantité

$$\text{gain: } H_t \cdot (B_{t+\Delta t} - B_t)$$

$(t \rightarrow t + \Delta t)$

$$\text{gain } 0 \rightarrow T$$

$$\sum_t \text{gain}(t, H_t)$$

$$\int_{\text{Rudby}}^{\text{FDE}} - \int_0^t = \int_{\text{Rudby}}^{\text{FDE}} \approx H_t (B_{t+\Delta t} - B_t)$$

$$S_{\text{Rudby}} \sim \Delta t \alpha_t$$

$$= \Delta t \sqrt{\Delta t}$$

$$\Delta t = \cancel{t} I$$

$$\sqrt{\Delta t} \mathcal{U}(\omega, \Delta t)$$

$$=$$

$$\beta t = 10^{-2}$$

$$\underline{10^{-2}}$$

$$10$$

$$dX_t = \alpha dt + \sigma dW_t$$

$$H_a^2 d\mu = d\langle X \rangle_u$$

$$\Delta t = 10^{-2}$$

$$\underline{10^{-2}}$$

$$\underline{10^{-1}}$$

$$H_a^2 dt = d\langle X \rangle_e$$

$$\Delta t = 10^{-6}$$

$$\underline{10^{-6}}$$

$$\underline{10^{-3}}$$

variation quadratique

$$d\langle X \rangle_{t \geq 0}$$

$$\text{Ex } X \approx B$$

$$d\langle X \rangle_e \approx dt.$$

$$\text{Ex. resistance EDS} \quad dX_t = \alpha dt + \beta dW_t \quad \alpha, \beta \approx cst$$

$$X_t = X_0 + \alpha t + \beta W_t$$

$$dX_t = \mu X_t dt + \sigma X_t dW_t$$

$$a(t, \text{fig}) = \mu \text{fig}$$

$$b(t, \text{fig}) = \sigma \text{fig}$$

$$\frac{dX_t}{X_t} = \mu dt + \sigma dW_t$$

$$X_t$$

Black-Scholes