Introduction à la statistique non-paramétrique 2019/20
M1 Math Université Paris Dauphine - PSL
G. Turinici

→ site cours pdm version recente poly

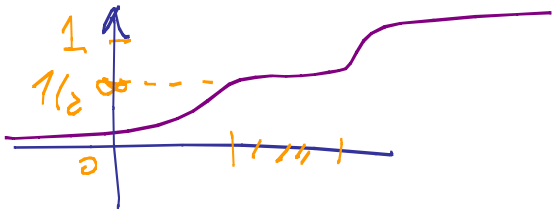
→ R → python

$f(t) = \int_{\mathbb{R}} \hat{f}(\omega)\, e^{i\omega x}\, d\omega$ ↓ param ?

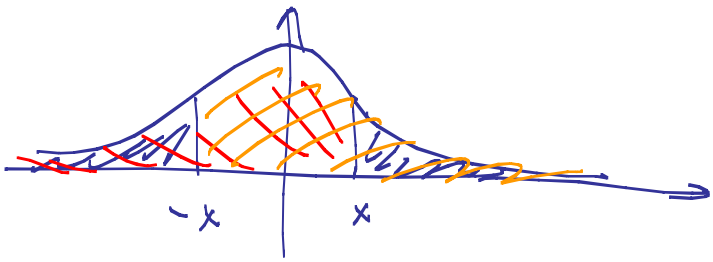
$f(t) = \sum_{n=1}^{\infty} a_n e^{inx}$ ↓ param ?

$\int_{\cdots} f(t)\, dx$ ↓ densité

Quantiles



Sym



$X \perp Y \quad \forall A \in \sigma(X), B \in \sigma(Y)$

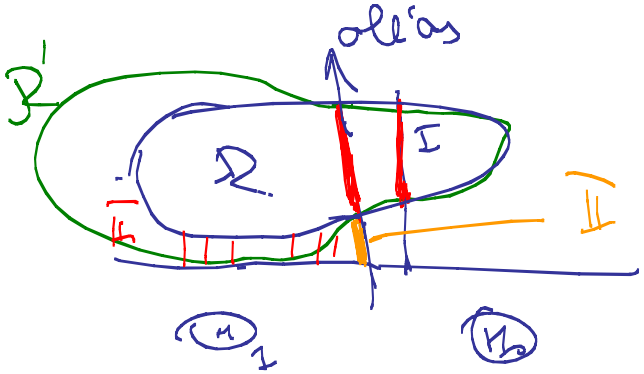
$\mathbb{P}(X \in A, Y \in B) = \mathbb{P}(X \in A)\, \mathbb{P}(Y \in B).$

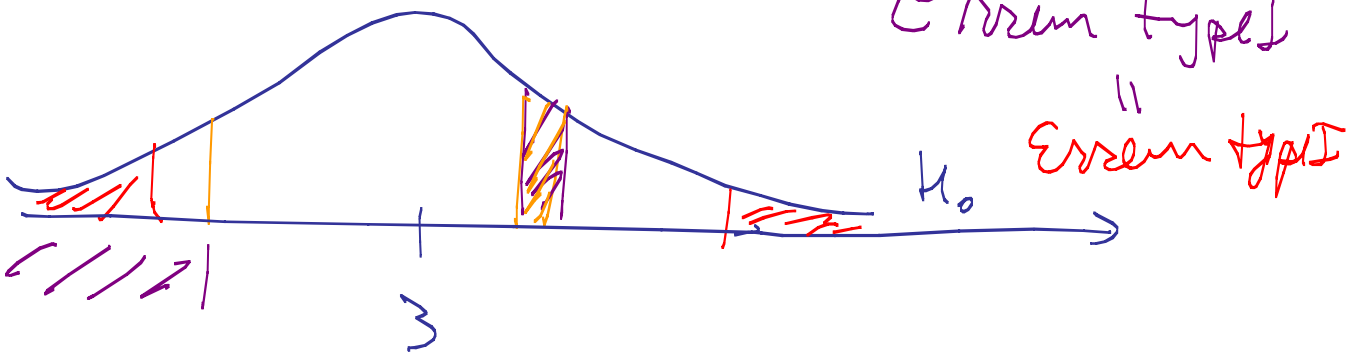
$X \to |X|$

$Y \to \mathbb{1}_{X>0}$

1.7 H_1 aussi noté H_a

Test





$$p(X) = \inf \{ \alpha \in [0,1] : \phi_\alpha(X) = 1 \}$$

$p(X) \leq \alpha$ on rejette H_0 au niveau α

$\Updownarrow$

$\inf \{ \alpha \mid \phi_\alpha(X) = 1 \} \leq \alpha$

$\exists \tilde{\alpha}$ $\quad \phi_{\tilde{\alpha}}(X) = 1 \qquad \tilde{\alpha} < \alpha$

$X \in R$ pour $\tilde{\alpha}$. $\quad \mathcal{R}_{\tilde{\alpha}}$

Question $\quad \tilde{\alpha} < \alpha \Rightarrow \mathcal{R}_{\tilde{\alpha}} \subset \mathcal{R}_{\alpha}$?

Prop 2.4 $\quad Y_k(\omega) = \mathbb{1}_{X_k \leq x}(\omega)$

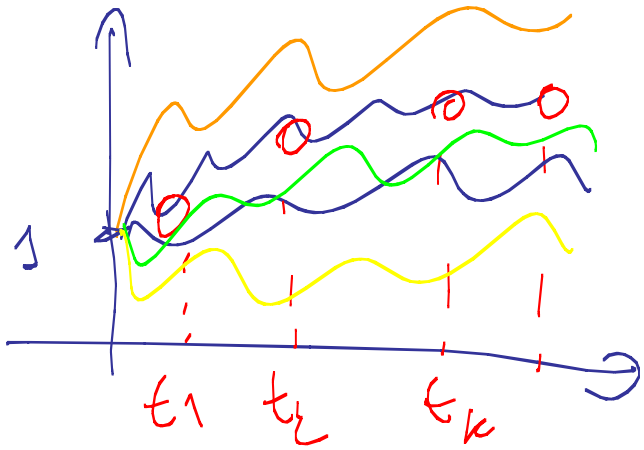
$$\mathbb{E}(Y_k) = \mathbb{E}\left(\mathbb{1}_{X_k \leq x}\right) = P(X_k \leq x)$$

$= F(x)$. $\quad Y_k \sim Be(F(x))$ $\quad \mathrm{Var}(Y_k) = F(x)(1-F(x))$

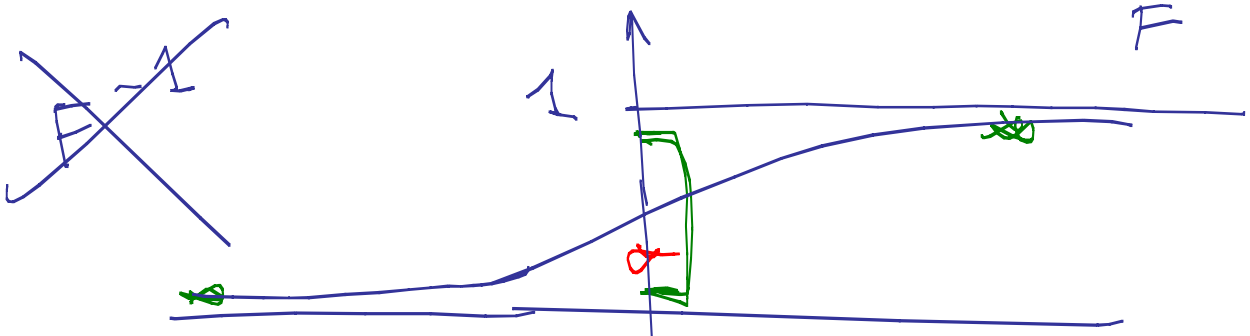
$$\hat{F}_n = \frac{\sum_{k=1}^{n} Y_k}{n}.$$

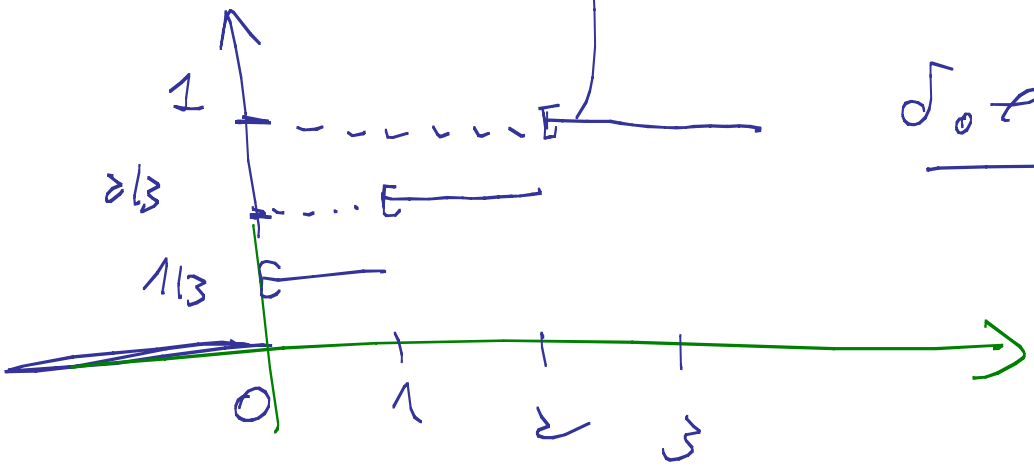
Question loi μ $\quad$ CDF : F

$X_1, \ldots, X_n, \ldots$ $\quad$ i.i.d.



$1 + W_t \sim \mathcal{N}(1, t)$

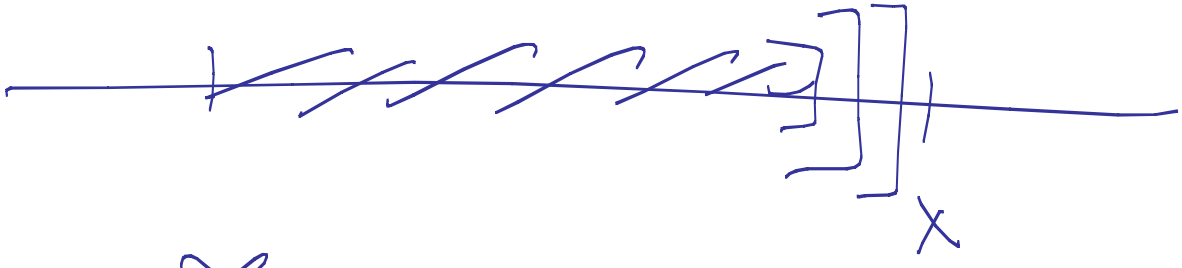




$$\frac{\delta_0 + \delta_1 + \delta_2}{3}$$

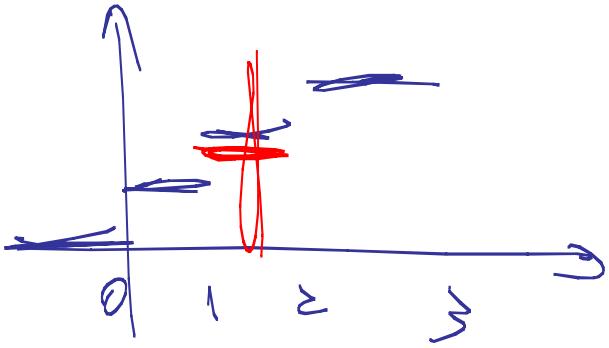
$q_{25\%} = 0$

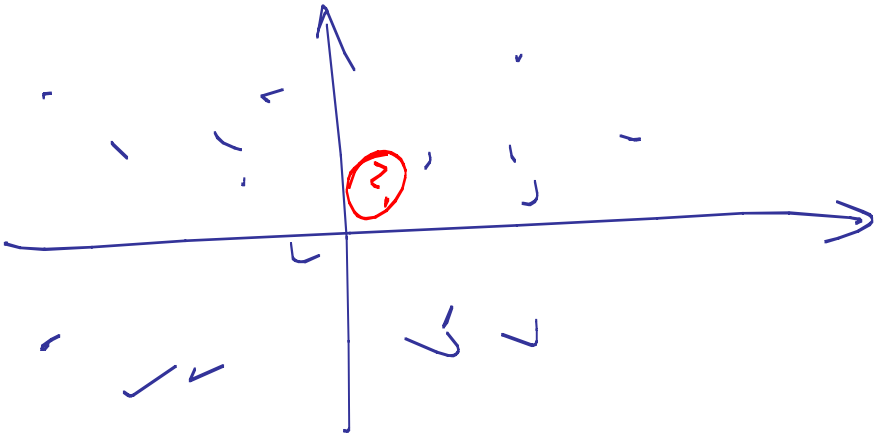
$q_{75\%} = 2$



$-\infty$

$\delta_0 + \delta_1 + \delta_2 + \delta_3$





$X : \Omega \to \mathbb{R}^2$

$\#\{X = m\}$

Test de Wilcoxon

Si X sym. diffuse (autour de sa médiane $=0$) alors l'aléa de X se compose de 2 parties indépendantes $\rightarrow |X|$

$\rightarrow \mathbb{1}_{X>0}$

$|X| \perp\!\!\!\perp \mathbb{1}_{X>0}$.

Thm 3.11 $\leq_{H_0}: W_n^+ \sim W_n^- \sim \sum_{k=1}^{n} k \cdot Y_k$ $\quad Y_k \sim Be(1/2)$ i.i.d.

Annonces $\rightarrow$ cours samedi

$\rightarrow$ partiel lundi $\rightarrow$ 1H manuscrite et individuel.

$\rightarrow$ feuille A4 recto-verso

$\rightarrow$ chap 1 et 2.